Worked Solutions

Edexcel C3 Paper K

1. $\dfrac{\cancel{(2x-3)}\cancel{(x+2)}}{\cancel{(2x-3)}(2x+3)} \times \dfrac{(x-1)}{(x+2)^{\cancel{2}}} = \dfrac{x-1}{(2x+3)(x+2)}$ **(5)**

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2. (*a*) $\text{fg}(x) = \dfrac{2}{x-3} + 2 = \dfrac{2x-4}{x-3}$, domain $x \in \mathbb{R},\ x \neq 3$

$\text{gf}(x) = \dfrac{2}{x+2-3} = \dfrac{2}{x-1}$, domain $x \in \mathbb{R},\ x \neq 1$ **(4)**

(*b*) $\dfrac{2x-4}{x-3} = \dfrac{2}{x-1} \Rightarrow 2x^2 - 6x + 4 = 2x - 6 \qquad x^2 - 4x + 5 = 0$

"$b^2 - 4ac < 0$" $\therefore$ no real solution. **(3)**

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3. (*a*) $\dfrac{\left(1-x^2\right) - x(-2x)}{(1-x^2)^2} = \dfrac{x^2+1}{(1-x^2)^2}$ **(3)**

(*b*) $x^2 \cdot \dfrac{1}{x} + 2x \cdot \ln x = x(1 + 2\ln x)$ **(3)**

(*c*) $\cos x \cdot e^{\sin x}$ **(3)**

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4. (*a*) $\dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin\theta}$

$\cos^2\theta \geq 0 \quad \because$ squared term

$\sin\theta > 0$ because $0 < \theta < 180°$ **(4)**

(*b*) $1 + \tan^2 x - 4\tan x + 2 = 0$

$\tan^2 x - 4\tan x + 3 = 0$

$(\tan x - 3)(\tan x - 1) = 0$

$$\therefore \begin{cases} \tan x = 3 \Rightarrow x = 71.6°(1\ \text{d.p.}), 251.6°(1\ \text{d.p.}) \\ \tan x = 1 \Rightarrow x = 45°, 225° \end{cases}$$ **(5)**

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5. (*a*) $\dfrac{dy}{dx} = x^3 + 3x^2 - 3$

Let $f(x) = x^3 + 3x^2 - 3$

$$\left.\begin{aligned} f(0) &= -3 \\ f(1) &= 1 \end{aligned}\right\} \text{sign change} \therefore \alpha$$

(*b*) $x^3 + 3x^2 - 3 = 0$

$x^3 + 3x^2 = 3$

$x^2(x+3) = 3$

$x^2 = \dfrac{3}{x+3}$

$x = \sqrt{\dfrac{3}{x+3}}$

(*c*) $x_1 = 0.8660,\ x_2 = 0.8809,\ x_3 = 0.8792,\ x_4 = 0.879$

(*d*) $\alpha = 0.879$

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6. (*a*) asymptote to $\ln(x+2)$ is $x = -2$, asymptote to l

(*b*) $B(-1, 0);\ C\left(\dfrac{1}{3}, 0\right)$

(*c*) $3x = x + 2$, A is $(1, \ln 3)$

(*d*) $\dfrac{dy}{dx} = \dfrac{1}{x}$ and $\dfrac{1}{x+2}$ gradients of curves at A : 1

$\tan\theta = \dfrac{1 - \frac{1}{3}}{1 + 1 \times \frac{1}{3}} = \dfrac{\frac{2}{3}}{\frac{4}{3}}$

$= \dfrac{1}{2}$

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7. (*a*) $2A = P + Q, 2B = P - Q$

$\sin(A + B) - \sin(A - B)$

$= \sin A \cos B + \cos A \sin B - \sin A \cos B + \cos A \sin B$

$= 2 \cos A \sin B$

$= 2 \cos\left(\frac{P+Q}{2}\right) \sin\left(\frac{P-Q}{2}\right)$ **(5)**

(*b*) $\sin 4\theta - \sin 2\theta + \cos 3\theta = 0$

$2 \cos 3\theta \sin\theta + \cos 3\theta = 0$

$\cos 3\theta(2 \sin\theta + 1) = 0$

$\cos 3\theta = 0 \Rightarrow 3\theta = 90, 270, 450, 630, 810, 990$

$\theta = 30, 90, 150, 210, 270, 330$

$\sin\theta = -\frac{1}{2} \Rightarrow \theta = 210, 330$ **(6)**

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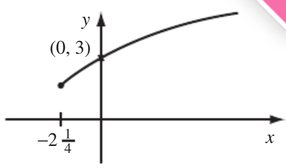
8. (*a*) $f(x) = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4}$ $\quad \therefore \; f(x) \geq$

(*b*) f^{-1} : (domain) $x \geq -2\frac{1}{4}$, (range) $f^{-1} \geq 1\frac{}{2}$

(*c*)



(*d*) gf: $x \mapsto \left|x^2 - 3x - 4\right|, x \in \mathbb{R}, x \geq \frac{3}{2}$

(*e*) either $x^2 - 3x - 4 = 6$

$x^2 - 3x - 10 = 0 \qquad (x - 5)(x + 2) = 0 \qquad x$

or $x^2 - 3x - 4 = -6 \qquad x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0 \qquad x = 2, x = 1$

solutions are $x = 2, 5 \quad \left(\text{since } x \geq \frac{3}{2}\right)$
